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GENERALIZED FRACTIONAL CALCULUS FORMULAS INVOLVING THE PRODUCT OF ALEPH-FUNCTION AND SRIVASTAVA POLYNOMIALS

DINESH KUMAR[A], R. K. GUPTA[B], B. S. SHAKTAWAT[C], AND JUNESANG CHOI[D,*]

Abstract. The object of this paper is to establish four formulas for the Saigo-Maeda fractional calculus operators associated with product of the Aleph-function and the general class of polynomials (Srivastava polynomials), which are expressed in terms of the Aleph-function of two variables. Since the involved fractional calculus operators, the Aleph-function, and the Srivastava polynomials are very general, the main results here can be specialized to yield a large number of formulas for fractional calculus operators involving various special functions and polynomials. Here some special cases are demonstrated.

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Keywords and phrases. Saigo-Maeda fractional calculus operators, Aleph-function of one and two variables, a general class of polynomials (Srivastava polynomials), I-function, H-function, Mittag-Leffler function, generalized Wright hypergeometric function ${}_p\Psi_q$

1. Introduction and Preliminaries

Throughout this paper, let $\mathbb{C}$, $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{N}$ be the sets of complex numbers, real and positive-real numbers, and positive integers, respectively, and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. Also let $\overline{1,n} := \{1, 2, \ldots, n\}$ for some $n \in \mathbb{N}$. The $\aleph$-function was introduced by Südland *et al.* [29, 30] and defined by means of Mellin-Barnes contour integrals (see also [2, 4, 11, 18, 19, 21, 22])

(1)
$$\aleph\,[z] = \aleph^{m,n}_{p_k,q_k,\tau_k;r}\left[z\,\middle|\begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k}\\(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right] := \frac{1}{2\pi i}\int_{\mathfrak{L}} \Omega^{m,n}_{p_k,q_k,\tau_k;r}\,(\xi)\; z^{-\xi}\,\mathrm{d}\xi,$$

for all $z \in \mathbb{C}\setminus\{0\}$, where $i=\sqrt{-1}$ and

(2)
$$\Omega^{m,n}_{p_k,q_k,\tau_k;r}\,(\xi) := \frac{\prod_{j=1}^{m}\Gamma\left(b_j+B_j\xi\right)\prod_{j=1}^{n}\Gamma\left(1-a_j-A_j\xi\right)}{\sum_{k=1}^{r}\tau_k\prod_{j=m+1}^{q_k}\Gamma\left(1-b_{jk}-B_{jk}\xi\right)\prod_{j=n+1}^{p_k}\Gamma\left(a_{jk}+A_{jk}\xi\right)},$$

Γ being the familiar Gamma function. The integration path $\mathfrak{L}=\mathfrak{L}_{i\gamma\infty}$ ($\gamma\in\mathbb{R}$) extends from $\gamma-i\infty$ to $\gamma+i\infty$ with indentations (if necessary), and is so described that the poles of $\Gamma\left(1-a_j-A_j\xi\right)$ ($j\in\overline{1,n}$) do not coincide with the poles of $\Gamma\left(b_j+B_j\xi\right)$ ($j\in\overline{1,m}$). The parameters p_k, $q_k\in\mathbb{N}_0$ satisfy the condition $0\le n\le p_k,\ 1\le m\le q_k$, and $\tau_k\in\mathbb{R}^+$ ($k\in\overline{1,r}$). The parameters $A_j, B_j, A_{jk}, B_{jk}\in\mathbb{R}^+$ and $a_j, b_j, a_{jk}, b_{jk}\in\mathbb{C}$. An empty product

* Corresponding author.

in (2) is interpreted as unity. The existence conditions for the integral (1) are given as follows:

$$\varphi_k \in \mathbb{R}^+, \quad |\arg(z)| < \frac{\pi}{2}\varphi_k; \tag{3}$$

$$\varphi_k \in \mathbb{R}^+ \cup \{0\}, \quad |\arg(z)| < \frac{\pi}{2}\varphi_k, \quad \Re(\zeta_k) + 1 < 0, \tag{4}$$

where

$$\varphi_k := \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_k \left(\sum_{j=n+1}^{p_k} A_{jk} + \sum_{j=m+1}^{q_k} B_{jk} \right), \tag{5}$$

$$\zeta_k := \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_k \left(\sum_{j=m+1}^{q_k} b_{jk} - \sum_{j=n+1}^{p_k} a_{jk} \right) + \frac{1}{2}\left(p_k - q_k\right). \tag{6}$$

Here $k \in \overline{1,r}$.

Remark 1. *The case* $\tau_k = 1$ $(k \in \overline{1,r})$ *in* (1) *reduces to the* I*-function* (*see* [16]). *Further special case* $r = 1$ *of the* I*-function is seen to become the familiar* H*- function* (*see* [8, 9]).

The ℵ-function of two variables which will be used in this paper is defined and represented by Saxena *et al.* [23] in the following manner:

(7)
$$\aleph[x,y] = \aleph^{o,n;m_1,n_1;m_2,n_2}_{p,q,p_k,q_k,\tau_i;p'_k,q'_k,\tau'_k;r} \left[\begin{array}{c|l} x & (a_j;\alpha_j,A_j)_{1,p};(c_j,C_j)_{1,n_1},\ldots,[\tau_j(c_j,C_j)]_{n_1+1,p_k};(e_j,E_j)_{1,n_2},\ldots,\left[\tau'_j(e_j,E_j)\right]_{n_2+1,p'_k} \\ y & (b_j;\beta_j,B_j)_{1,q};(d_j,D_j)_{1,m_1},\ldots,[\tau_j(d_j,D_j)]_{m_1+1,q_k};(f_j,F_j)_{1,m_2},\ldots,\left[\tau'_j(f_j,F_j)\right]_{m_2+1,q'_k} \end{array} \right]$$
$$= \left(\frac{1}{2\pi i}\right)^2 \int_{\mathfrak{L}_1}\int_{\mathfrak{L}_2} \phi(s,\xi)\,\theta_1(s)\,\theta_2(\xi) x^{-s}\, y^{-\xi}\, \mathrm{d}s\ \mathrm{d}\xi,$$

for all $x,\ y \in \mathbb{C} \setminus \{0\}$, where $i = \sqrt{-1}$ and

$$\phi(s,\xi) := \frac{\prod_{j=1}^{n} \Gamma\left(1 - a_j - \alpha_j s - A_j \xi\right)}{\prod_{j=n+1}^{p} \Gamma\left(a_j + \alpha_j s + A_j \xi\right) \prod_{j=1}^{q} \Gamma\left(1 - b_j - \beta_j s - B_j \xi\right)}, \tag{8}$$

(9)
$$\theta_1(s) := \Omega^{m_1,n_1}_{p_k,q_k,\tau_k;r}(s) = \frac{\prod_{j=1}^{m_1} \Gamma\left(d_j + D_j s\right) \prod_{j=1}^{n_1} \Gamma\left(1 - c_j - C_j s\right)}{\sum_{k=1}^{r} \tau_k \prod_{j=m_1+1}^{q_k} \Gamma\left(1 - d_{jk} - D_{jk} s\right) \prod_{j=n_1+1}^{p_k} \Gamma\left(c_{jk} + C_{jk} s\right)},$$

(10)
$$\theta_2(\xi) := \Omega^{m_2,n_2}_{p'_k,q'_k,\tau'_k;r}(\xi) = \frac{\prod_{j=1}^{m_2} \Gamma\left(f_j + F_j \xi\right) \prod_{j=1}^{n_2} \Gamma\left(1 - e_j + E_j \xi\right)}{\sum_{k=1}^{r} \tau'_k \prod_{j=m_2+1}^{q'_k} \Gamma\left(1 - f_{jk} - F_{jk} \xi\right) \prod_{j=n_2+1}^{p'_k} \Gamma\left(e_{jk} + E_{jk} \xi\right)}.$$

Here all involved parameters are similarly restricted as in the ℵ-function of one variable (1).

The general class of polynomials (Srivastava polynomials) $S_n^m[x]$ is defined and represented as follows ([24, p.1, eq.1])

$$S_n^m[x] = \sum_{k=0}^{[n/m]} \frac{(-n)_{mk}}{k!} A_{n,k}\, x^k \quad (n \in \mathbb{N}_0,\, m \in \mathbb{N}), \tag{11}$$

where the coefficients $A_{n,k}$ $(n,\, k \in \mathbb{N}_0)$ are arbitrary constants, real or complex. On suitably specializing the coefficients $A_{n,k}$, the family of polynomials $S_n^m[x]$ yields a number of known polynomials as its special cases (see [28]).

For a more detailed account of fractional calculus operators and the Srivastava polynomials, the interested reader may refer to many recent works such as [1, 2, 3, 5, 6, 17, 18, 25, 27]. In particular, Saxena and Kumar [18] have established generalized fractional calculus formulas involving the product of ℵ-function of one variable and a general class of polynomials (Srivastava polynomials). Choi and Kumar [2] presented fractional calculus formulas for product of ℵ-function and a general class of multivariable polynomials which are expressed in terms of the ℵ-function of one variable. Here, in this paper, we aim to establish four formulas for the Saigo-Maeda fractional calculus operators associated with product of the Aleph-function and the general class of polynomials (Srivastava polynomials), which are expressed in terms of the Aleph-function of two variables. Among a large numbers of special cases of our main results, only some of which are demonstrated.

It is remarked in passing that the formulas presented here are different from those earlier related results due mainly to their expressions in terms the ℵ-function of *two variables*.

2. Generalized Fractional Calculus Operators

Let $\alpha,\, \alpha',\, \beta,\, \beta',\, \gamma \in \mathbb{C}$, $x \in \mathbb{R}^+$ and $\Re(\gamma) > 0$. Then the generalized fractional integral operators involving Appell's function (Horn's function) F_3 were introduced by Marichev [7] and later extended and investigated by Saigo and Maeda [14] as follows (see [14, p. 393, Eqs. (4.12) and (4.13)]):

$$\begin{aligned}&\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x)\\ &\quad = \frac{x^{-\alpha}}{\Gamma(\gamma)} \int_0^x t^{-\alpha'} (x-t)^{\gamma-1}\, F_3\left(\alpha, \alpha', \beta, \beta'; \gamma;\, 1-\frac{t}{x}, 1-\frac{x}{t}\right) f(t)\, \mathrm{d}t\end{aligned} \tag{12}$$

and

$$\begin{aligned}&\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} f\right)(x)\\ &\quad = \frac{x^{-\alpha'}}{\Gamma(\gamma)} \int_x^\infty t^{-\alpha} (t-x)^{\gamma-1}\, F_3\left(\alpha, \alpha', \beta, \beta'; \gamma;\, 1-\frac{x}{t}, 1-\frac{t}{x}\right) f(t)\, \mathrm{d}t.\end{aligned} \tag{13}$$

These operators (12) and (13) reduce to the Saigo fractional integral operators as follows:

$$\left(I_{0+}^{\alpha,0,\beta,\beta',\gamma} f\right)(x) = \left(I_{0+}^{\gamma,\alpha-\gamma,-\beta} f\right)(x) \quad \left(\alpha,\, \beta,\, \gamma \in \mathbb{C};\; x \in \mathbb{R}^+\right) \tag{14}$$

and

$$\left(I_{-}^{\alpha,0,\beta,\beta',\gamma}f\right)(x) = \left(I_{-}^{\gamma,\alpha-\gamma,-\beta}f\right)(x) \quad \left(\alpha,\,\beta,\,\gamma \in \mathbb{C};\; x \in \mathbb{R}^{+}\right). \tag{15}$$

The generalized fractional derivative operators [14] involving Appell's function F_3 as a kernel are defined by

$$\begin{aligned}\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x) &= \left(I_{0+}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma}f\right)(x) \quad (\Re(\gamma)>0)\\ &= \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{k}\left(I_{0+}^{-\alpha',-\alpha,-\beta'+k,-\beta,-\gamma+k}f\right)(x) \quad (\Re(\gamma)>0;\; k=[\Re(\gamma)]+1)\end{aligned} \tag{16}$$

and

$$\begin{aligned}\left(D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}f\right)(x) &= \left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta,-\gamma}f\right)(x) \quad (\Re(\gamma)>0)\\ &= \left(-\frac{\mathrm{d}}{\mathrm{d}x}\right)^{k}\left(I_{-}^{-\alpha',-\alpha,-\beta',-\beta+k,-\gamma+k}f\right)(x) \quad (\Re(\gamma)>0;\; k=[\Re(\gamma)]+1).\end{aligned} \tag{17}$$

These operators (16) and (17) reduce to the Saigo fractional derivative operators as follows (see [12, 20, 21]):

$$\left(D_{0+}^{0,\alpha',\beta,\beta',\gamma}f\right)(x) = \left(D_{0+}^{\gamma,\alpha'-\gamma,\beta'-\gamma}f\right)(x) \quad \left(\Re(\gamma)>0;\; x\in\mathbb{R}^{+}\right) \tag{18}$$

and

$$\left(D_{-}^{0,\alpha',\beta,\beta',\gamma}f\right)(x) = \left(D_{-}^{\gamma,\alpha'-\gamma,\beta'-\gamma}f\right)(x) \quad \left(\Re(\gamma)>0;\; x\in\mathbb{R}^{+}\right). \tag{19}$$

We recall the following known formulas (see [14, p. 394, Eqs.(4.18) and (4.19)]):

$$\begin{aligned}&\left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\right)(x)\\ &\quad= \frac{\Gamma(\rho)\,\Gamma(\rho+\gamma-\alpha-\alpha'-\beta)\,\Gamma(\rho+\beta'-\alpha')}{\Gamma(\rho+\gamma-\alpha-\alpha')\,\Gamma(\rho+\gamma-\alpha'-\beta)\,\Gamma(\rho+\beta')}\,x^{\rho-\alpha-\alpha'+\gamma-1}\\ &\left(\Re(\gamma)>0,\;\Re(\rho)>\max\left\{0,\Re\left(\alpha+\alpha'+\beta-\gamma\right),\Re\left(\alpha'-\beta'\right)\right\},\; x\in\mathbb{R}^{+}\right)\end{aligned} \tag{20}$$

and

$$\begin{aligned}&\left(I_{-}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\rho-1}\right)(x)\\ &= \frac{\Gamma(1+\alpha+\alpha'-\gamma-\rho)\,\Gamma(1+\alpha+\beta'-\gamma-\rho)\,\Gamma(1-\beta-\rho)}{\Gamma(1-\rho)\,\Gamma(1+\alpha+\alpha'+\beta'-\gamma-\rho)\,\Gamma(1+\alpha-\beta-\rho)}\,x^{\rho-\alpha-\alpha'+\gamma-1}\\ &\left(\Re(\gamma)>0,\;\Re(\rho)<1+\min\left\{\Re(-\beta),\Re\left(\alpha+\alpha'-\gamma\right),\Re\left(\alpha+\beta'-\gamma\right)\right\},\; x\in\mathbb{R}^{+}\right).\end{aligned} \tag{21}$$

3. Generalized Fractional Integral Formulas

Here we establish two fractional integral formulas for the product of $\aleph$-function and Srivastava polynomials asserted by Theorems 3.1 and 3.4.

Theorem 3.1. *Let α, α', β, β', γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ with $\Re(\gamma) > 0$. Also let λ, $\sigma \in \mathbb{R}^+$. Then the following formula holds true:*

(22)

$$\begin{aligned}
&\Bigg\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\Bigg(t^{\mu-1}(b-at)^{-\eta} S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right] \\
&\times \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^{\sigma}(b-at)^{-\upsilon}\,\middle|\, \begin{matrix} (a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k} \\ (b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right]\Bigg)\Bigg\}(x) \\
&= b^{-\eta}x^{\mu-\alpha-\alpha'+\gamma-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right) \\
&\times \aleph_{4,4;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,4;m,n;1,0}\left[\begin{matrix} zb^{-\upsilon}x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\, \begin{matrix} E_1;(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k};- \\ E_2;(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k};(0,-1)\end{matrix}\right],
\end{aligned}$$

where E_1 and E_2 are the following horizontal arrays:

$$\begin{aligned}
E_1 := &(1-\eta-\delta l;\upsilon,-1),\,(1-\mu-\lambda l;\sigma,-1),\,\big(1-\mu-\gamma+\alpha+\alpha'+\beta-\lambda l;\sigma,-1\big), \\
&\big(1-\mu+\alpha'-\beta'-\lambda l;\sigma,-1\big)
\end{aligned}$$

and

$$\begin{aligned}
E_2 := &(1-\eta-\delta l;\upsilon,0),\,\big(1-\mu-\gamma+\alpha+\alpha'-\lambda l;\sigma,-1\big),\,\big(1-\mu-\gamma+\alpha'+\beta-\lambda l;\sigma,-1\big), \\
&\big(1-\mu-\beta'-\lambda l;\sigma,-1\big).
\end{aligned}$$

The following conditions are also satisfied:

$$\Re(\mu)+\sigma\min_{1\le j\le m}\left(\frac{\Re(b_j)}{B_j}\right) > \max\big\{0,\Re\left(\alpha+\alpha'+\beta-\gamma\right),\Re\left(\alpha'-\beta'\right)\big\};$$

$$\Re(\eta)+\upsilon\min_{1\le j\le m}\left(\frac{\Re(b_j)}{B_j}\right) > \max\big\{0,\Re\left(\alpha+\alpha'+\beta-\gamma\right),\Re\left(\alpha'-\beta'\right)\big\};$$

and $\left|\frac{a}{b}x\right| < 1$. The conditions given in (3)-(6) *are further satisfied.*

Proof. Let $\mathcal{I}_1$ be the left-hand side of (22). Then express the Srivastava polynomials in the defining series and the $\aleph$-function in terms of the Mellin-Barnes contour to give

$$\begin{aligned}
\mathcal{I}_1 = \Bigg\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\Bigg(&\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\frac{1}{2\pi i}\int_{\mathfrak{L}_1}\Omega_{p_k,q_k,\tau_k;r}^{m,n}(s)\,z^{-s}\,\mathrm{d}s \\
&\times b^{-\eta-\delta\ell+\nu s}\left(1-\frac{a}{b}t\right)^{-(\eta+\delta\ell-\nu s)}t^{\mu+\lambda\ell-\sigma s-1}\Bigg)\Bigg\}(x).
\end{aligned}$$

Next using the following binomial expansion for $(b-ax)^{-\gamma}$:

$$(b-ax)^{-\gamma} = b^{-\gamma}\sum_{s=0}^{\infty}\frac{(\gamma)_s}{s!}\left(\frac{ax}{b}\right)^s, \qquad \left|\frac{ax}{b}\right| < 1 \tag{23}$$

and interchanging the order of integrals and summations, after a little simplification, we obtain

$$\begin{aligned}\mathcal{I}_1 &= b^{-\eta} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} \left(\frac{1}{2\pi i}\right)^2 \int_{\mathfrak{L}_1} \int_{\mathfrak{L}_2} \Omega_{p_k,q_k,\tau_k;r}^{m,n}(s) \\ &\times \left(zb^{-\nu}\right)^{-s} \left(-\frac{a}{b}\right)^{\xi} \frac{\Gamma(\eta+\delta\ell-\nu s+\xi)}{\Gamma(\eta+\delta\ell-\nu s)\,\Gamma(1+\xi)}\, \mathrm{d}s\, \mathrm{d}\xi \left(I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} t^{\mu+\lambda\ell-\sigma s+\xi-1}\right)(x) \\ &= b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \sum_{j=1}^{r} \tau_j' \left(\frac{1}{2\pi i}\right)^2 \\ &\times \int_{\mathfrak{L}_1} \Omega_{p_k,q_k,\tau_k;r}^{m,n}(s) \left(zb^{-\nu}x^{\sigma}\right)^{-s} \mathrm{d}s \int_{\mathfrak{L}_2} \frac{1}{\sum_{j=1}^{r} \tau_j' \,\Gamma(1+\xi)} \left\{\left(-\frac{a}{b}x\right)^{-1}\right\}^{-\xi} \mathrm{d}\xi \\ &\times \frac{\Gamma(\eta+\delta\ell-\nu s+\xi)\, \Gamma(\mu+\lambda\ell-\sigma s+\xi)}{\Gamma(\eta+\delta\ell-\nu s)\, \Gamma(\mu+\lambda\ell-\sigma s+\xi+\gamma-\alpha-\alpha')} \\ &\times \frac{\Gamma(\mu+\lambda\ell-\sigma s+\xi+\gamma-\alpha-\alpha'-\beta)}{\Gamma(\mu+\lambda\ell-\sigma s+\xi+\gamma-\alpha'-\beta)} \frac{\Gamma(\mu+\lambda\ell-\sigma s+\xi-\alpha'+\beta')}{\Gamma(\mu+\lambda\ell-\sigma s+\xi+\beta')}.\end{aligned} \tag{24}$$

By re-interpreting the Mellin-Barnes contour integral in terms of the ℵ-function of two variables given in (7), after a little simplification, $\mathcal{I}_1$ is seen to lead to the desired right-hand side of (22). This completes the proof. □

In view of the relation (14), we get (presumably) new formulas concerning the left-sided Saigo fractional integral operator (see [12, 13, 14]) in the following corollary.

Corollary 3.2. *Let α, β, γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ with $\Re(\alpha) > 0$. Also let λ, $\sigma \in \mathbb{R}^+$. Then the following formula holds true:*

$$\begin{aligned}&\Bigg\{I_{0+}^{\alpha,\beta,\gamma}\Bigg(t^{\mu-1}(b-at)^{-\eta} S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right] \\ &\quad\times \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^{\sigma}(b-at)^{-\upsilon} \,\middle|\, \begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k} \\ (b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right]\Bigg)\Bigg\}(x) \\ &\quad= b^{-\eta}x^{\mu-\beta-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \left(\sum_{j=1}^{r}\tau_j'\right) \\ &\times \aleph_{3,3;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,3;m,n;1,0}\left[\begin{matrix} zb^{-\upsilon}x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1}\end{matrix} \,\middle|\, \begin{matrix} E_1';(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k};- \\ E_2';(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k};(0,-1)\end{matrix}\right],\end{aligned} \tag{25}$$

where E_1' and E_2' are horizontal arrays given as follows:

$$E_1' := (1-\eta-\delta\ell;\upsilon,-1),\ (1-\mu-\lambda\ell;\sigma,-1),\ (1-\mu-\lambda\ell+\beta-\gamma;\sigma,-1)$$

and

$$E_2' := (1-\eta-\delta\ell;\upsilon,0),\ (1-\mu-\lambda\ell+\beta;\sigma,-1)\ (1-\mu-\lambda\ell-\alpha-\gamma;\sigma,-1).$$

Also the following conditions are satisfied:

$$\Re(\mu) + \sigma \min_{1 \le j \le m} \left(\frac{\Re(b_j)}{B_j} \right) > \max\{0, \Re(\beta - \gamma)\};$$

$$\Re(\eta) + \upsilon \min_{1 \le j \le m} \left(\frac{\Re(b_j)}{B_j} \right) > \max\{0, \Re(\beta - \gamma)\};$$

and $\left|\frac{a}{b}x\right| < 1$. *The conditions which are adjusted from those in* (3)-(6) *are further satisfied.*

Further if we set $\beta = -\alpha$ in (25), we obtain a (presumably) new formula regarding the left-sided Riemann-Liouville fractional integral operator (see [10, 12]) asserted by the following corollary.

Corollary 3.3. *Let* α, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\alpha) > 0$. *Also let* λ, $\sigma \in \mathbb{R}^+$. *Then the following formula holds true:*

(26)

$$\begin{aligned} &\Bigg\{ I_{0+}^{\alpha} \Bigg(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \\ &\quad \times \aleph_{p_k, q_k, \tau_k; r}^{m,n} \left[z\, t^{\sigma} (b-at)^{-\upsilon} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j (a_j, A_j)]_{n+1, p_k} \\ (b_j, B_j)_{1,m}, \ldots, [\tau_j (b_j, B_j)]_{m+1, q_k} \end{matrix} \right] \Bigg) \Bigg\} (x) \\ &= b^{-\eta} x^{\mu+\alpha-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \left(\sum_{j=1}^{r} \tau_j' \right) \\ &\times \aleph_{2,2;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,2;m,n;1,0} \left[\begin{matrix} z b^{-\upsilon} x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1} \end{matrix} \,\middle|\, \begin{matrix} E_1''; (a_j, A_j)_{1,n}, \ldots, [\tau_j (a_j, A_j)]_{n+1,p_k}; - \\ E_2''; (b_j, B_j)_{1,m}, \ldots, [\tau_j (b_j, B_j)]_{m+1,q_k}; (0, -1) \end{matrix} \right], \end{aligned}$$

where E_1'' *and* E_2'' *are horizontal arrays given as follows:*

$$E_1'' = (1 - \eta - \delta\ell; \upsilon, -1), (1 - \mu - \lambda\ell; \sigma, -1)$$

and

$$E_2'' = (1 - \eta - \delta\ell; \upsilon, 0), (1 - \mu - \lambda\ell - \alpha; \sigma, -1).$$

Also the other conditions necessary for this formula are easily derivable from those in Corollary 3.2.

Theorem 3.4. *Let* α, α', β, β', γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\gamma) > 0$. *Also let* λ, $\sigma \in \mathbb{R}^+$. *Then the following formula holds true:*

(27)

$$\begin{aligned} &\Bigg\{ I_{-}^{\alpha,\alpha',\beta,\beta',\gamma} \Bigg(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \\ &\quad \times \aleph_{p_k, q_k, \tau_k; r}^{m,n} \left[z\, t^{\sigma} (b-at)^{-\upsilon} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j (a_j, A_j)]_{n+1, p_k} \\ (b_j, B_j)_{1,m}, \ldots, [\tau_j (b_j, B_j)]_{m+1, q_k} \end{matrix} \right] \Bigg) \Bigg\} (x) \\ &= b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \left(\sum_{j=1}^{r} \tau_j' \right) \\ &\times \aleph_{4,4;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,4;m,n;1,0} \left[\begin{matrix} z b^{-\upsilon} x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1} \end{matrix} \,\middle|\, \begin{matrix} F_1; (a_j, A_j)_{1,n}, \ldots, [\tau_j (a_j, A_j)]_{n+1,p_k}; - \\ F_2; (b_j, B_j)_{1,m}, \ldots, [\tau_j (b_j, B_j)]_{m+1,q_k}; (0, -1) \end{matrix} \right], \end{aligned}$$

where F_1 *and* F_2 *are horizontal arrays given by*

$$F_1 = (1-\eta-\delta\ell;\upsilon,-1)\,,\, \left(\mu+\lambda\ell-\alpha-\alpha'+\gamma;-\sigma,1\right),\\ \left(\mu+\lambda\ell-\alpha-\beta'+\gamma;-\sigma,1\right),\, (\mu+\lambda\ell+\beta;-\sigma,1)$$

and

$$F_2 = (1-\eta-\delta\ell;\upsilon,0)\,,\, (\mu+\lambda\ell;-\sigma,1)\,,\, \left(\mu+\lambda\ell-\alpha-\alpha'-\beta'+\gamma;-\sigma,1\right),\\ (\mu+\lambda\ell-\alpha+\beta;-\sigma,1)\,.$$

Also the following conditions are satisfied:

$$\Re(\mu)+\sigma\max_{1\le j\le n}\left(\frac{\Re(a_j)-1}{A_j}\right) < 1+\min\left\{\Re(-\beta)\,,\Re\left(\alpha+\alpha'-\gamma\right),\Re\left(\alpha+\beta'-\gamma\right)\right\};$$

$$\Re(\eta)+\upsilon\max_{1\le j\le n}\left(\frac{\Re(a_j)-1}{A_j}\right) < 1+\min\left\{\Re(-\beta)\,,\Re\left(\alpha+\alpha'-\gamma\right),\Re\left(\alpha+\beta'-\gamma\right)\right\};$$

and $\left|\frac{a}{b}x\right| < 1$*. Further the other conditions are derivable from those in* (3)-(6)*.*

Proof. The proof would run in parallel with that of Theorem 3.1. So its detailed account is omitted. □

In view of the relation (15), we get an integral formula concerning the right-sided Saigo fractional integral operator (see [12, 13, 14]) as in the following corollary.

Corollary 3.5. *Let* α, β, γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\alpha) > 0$*. Also let* λ, $\sigma \in \mathbb{R}^+$*. Then the following formula holds true:*

(28)

$$\left\{I_{-}^{\alpha,\beta,\gamma}\left(t^{\mu-1}(b-at)^{-\eta}\,S_n^m\left[t^\lambda(b-at)^{-\delta}\right]\right.\right.\\ \left.\left.\times\ \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^\sigma(b-at)^{-\upsilon}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n}\,,\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k}\\(b_j,B_j)_{1,m}\,,\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right]\right)\right\}(x)\\ = b^{-\eta}\,x^{\mu-\beta-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}\,x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right)\\ \times\ \aleph_{3,3;p_k,q_k,\tau_k;0,1,\tau_k';r}^{\,0,3;m,n;1,0}\left[\begin{matrix}zb^{-\upsilon}x^\sigma\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}F_1';(a_j,A_j)_{1,n}\,,\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k}\,;-\\F_2';(b_j,B_j)_{1,m}\,,\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\,;(0,-1)\end{matrix}\right],$$

where F_1' *and* F_2' *are horizontal arrays given as follows:*

$$F_1' = (1-\eta-\delta\ell;\upsilon,-1)\,,\,(\mu+\lambda\ell-\beta;-\sigma,1)\,,\,(\mu+\lambda\ell-\gamma;-\sigma,1)\,;$$

$$F_2' = (1-\eta-\delta\ell;\upsilon,0)\,,\,(\mu+\lambda\ell;-\sigma,1)\ (\mu+\lambda\ell-\alpha-\beta-\gamma;-\sigma,1)\,;$$

and $\left|\frac{a}{b}x\right| < 1$*. Further the other conditions are derivable from those in* (3)-(6)*.*

Further if we set $\beta = -\alpha$ in (28), we obtain a formula concerning the right-sided Riemann-Liouville fractional integral operator as in the following corollary (see [10, 12]).

Corollary 3.6. *Let α, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ with $\Re(\alpha) > 0$. Also let λ, $\sigma \in \mathbb{R}^+$. Then the following formula holds true:*

(29)

$$\Bigg\{ I_-^{\alpha} \Bigg(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \times \aleph_{p_k,q_k,\tau_k;r}^{m,n} \left[z\, t^{\sigma} (b-at)^{-\upsilon} \,\middle|\, \begin{matrix} (a_j,A_j)_{1,n}, \ldots, [\tau_j(a_j,A_j)]_{n+1,p_k} \\ (b_j,B_j)_{1,m}, \ldots, [\tau_j(b_j,B_j)]_{m+1,q_k} \end{matrix} \right] \Bigg) \Bigg\} (x)$$

$$= b^{-\eta} x^{\mu+\alpha-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \left(\sum_{j=1}^{r} \tau_j' \right)$$

$$\times \aleph_{2,2;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,2;m,n;1,0} \left[\begin{matrix} zb^{-\upsilon}x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1} \end{matrix} \,\middle|\, \begin{matrix} F_1''; (a_j,A_j)_{1,n}, \ldots, [\tau_j(a_j,A_j)]_{n+1,p_k}; — \\ F_2''; (b_j,B_j)_{1,m}, \ldots, [\tau_j(b_j,B_j)]_{m+1,q_k}; (0,-1) \end{matrix} \right],$$

where F_1'' and F_2'' are horizontal arrays given as follows:

$$F_1'' = (1-\eta-\delta\ell;\upsilon,-1), (\mu+\lambda\ell+\alpha;-\sigma,1);\; F_2'' = (1-\eta-\delta\ell;\upsilon,0), (\mu+\lambda\ell;-\sigma,1);$$

and $\left|\frac{a}{b}x\right| < 1$. Further the other conditions are derivable from those of Corollary 3.5.

4. Generalized Fractional Derivative Formulas

Here we establish two fractional derivative formulas for the product of the $\aleph$-function and the general class of polynomials (Srivastava polynomials) asserted by Theorems 4.1 and 4.4.

Theorem 4.1. *Let α, α', β, β', γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ with $\Re(\gamma) > 0$. Also let λ, $\sigma \in \mathbb{R}^+$. Then the following formula holds true:*

(30)

$$\Bigg\{ D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} \Bigg(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \times \aleph_{p_k,q_k,\tau_k;r}^{m,n} \left[z\, t^{\sigma} (b-at)^{-\upsilon} \,\middle|\, \begin{matrix} (a_j,A_j)_{1,n}, \cdots, [\tau_j(a_j,A_j)]_{n+1,p_k} \\ (b_j,B_j)_{1,m}, \cdots, [\tau_j(b_j,B_j)]_{m+1,q_k} \end{matrix} \right] \Bigg) \Bigg\} (x)$$

$$= b^{-\eta} x^{\mu+\alpha+\alpha'-\gamma-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \left(\sum_{j=1}^{r} \tau_j' \right)$$

$$\times \aleph_{4,4;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,4;m,n;1,0} \left[\begin{matrix} zb^{-\upsilon}x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1} \end{matrix} \,\middle|\, \begin{matrix} V_1; (a_j,A_j)_{1,n}, \cdots, [\tau_j(a_j,A_j)]_{n+1,p_k}; — \\ V_2; (b_j,B_j)_{1,m}, \cdots, [\tau_j(b_j,B_j)]_{m+1,q_k}; (0,-1) \end{matrix} \right],$$

where V_1 and V_2 are the following horizontal arrays:

$$V_1 = (1-\eta-\delta\ell;\upsilon,-1), (1-\mu-\lambda\ell;\sigma,-1), \left(1-\mu-\lambda\ell-\alpha-\alpha'-\beta'+\gamma;\sigma,-1\right), (1-\mu-\lambda\ell-\alpha+\beta;\sigma,-1)$$

and

$$V_2 = (1-\eta-\delta\ell;\upsilon,0), \left(1-\mu-\lambda\ell-\alpha-\alpha'+\gamma;\sigma,-1\right), \left(1-\mu-\lambda\ell-\alpha-\beta'+\gamma;\sigma,-1\right), (1-\mu-\lambda\ell+\beta;\sigma,-1).$$

Also the following conditions are satisfied:

$$\Re(\mu)+\sigma\min_{1\leq j\leq m}\left(\frac{\Re(b_j)}{B_j}\right)+\max\left\{0,\Re\left(\alpha-\beta\right),\Re\left(\alpha+\alpha'+\beta'-\gamma\right)\right\}>0;$$

$$\Re(\eta)+\upsilon\min_{1\leq j\leq m}\left(\frac{\Re(b_j)}{B_j}\right)+\max\left\{0,\Re\left(\alpha-\beta\right),\Re\left(\alpha+\alpha'+\beta'-\gamma\right)\right\}>0;$$

and $\left|\frac{a}{b}x\right|<1$. *Further the other conditions are derivable from those in* (3)-(6).

Proof. Let $\mathcal{D}_1$ be the left-hand side of (30). Then expressing the Srivastava polynomials as the defining series and the ℵ-function in terms of Mellin-Barnes contour integral, we find

$$\begin{aligned}\mathcal{D}_1=&\left\{D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\frac{1}{2\pi i}\int_{\mathfrak{L}_1}\Omega_{p_k,q_k,\tau_k;r}^{m,n}(s)\,z^{-s}\,\mathrm{d}s\right.\right.\\&\left.\left.\times\, b^{-\eta-\delta\ell+\nu s}\left(1-\frac{a}{b}t\right)^{-(\eta+\delta\ell-\nu s)}t^{\mu+\lambda\ell-\sigma s-1}\right)\right\}(x).\end{aligned}\tag{31}$$

By using the binomial expansion (23) in (31), we can express the resulting identity in terms of Mellin-Barnes contour integral (see Srivastava *et al.* [26]). Then interchanging the order of derivative and summations, after a little simplification, we obtain

$$\begin{aligned}\mathcal{D}_1\;=&\;b^{-\eta}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}\left(\frac{1}{2\pi i}\right)^2\int_{\mathfrak{L}_1}\int_{\mathfrak{L}_2}\Omega_{p_k,q_k,\tau_k;r}^{m,n}(s)\\&\times\left(zb^{-\nu}\right)^{-s}\left(-\frac{a}{b}\right)^{\xi}\frac{\Gamma\left(\eta+\delta\ell-\nu s+\xi\right)}{\Gamma\left(\eta+\delta\ell-\nu s\right)\Gamma\left(1+\xi\right)}\,\mathrm{d}s\,\mathrm{d}\xi\,\left(D_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}t^{\mu+\lambda\ell-\sigma s+\xi-1}\right)(x)\,.\end{aligned}\tag{32}$$

By taking into account the relation (16), we have

$$\mathcal{D}_1=b^{-\eta}\,x^{\mu+\alpha+\alpha'-\gamma-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}\,x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right)\left(\frac{1}{2\pi i}\right)^2$$

$$\times\int_{\mathfrak{L}_1}\Omega_{p_k,q_k,\tau_k;r}^{m,n}(s)\,\left(zb^{-\nu}x^{\sigma}\right)^{-s}\mathrm{d}s\int_{\mathfrak{L}_2}\frac{1}{\sum_{j=1}^{r}\tau_j'\,\Gamma\left(1+\xi\right)}\left\{\left(-\frac{a}{b}x\right)^{-1}\right\}^{-\xi}\mathrm{d}\xi$$

$$\times\frac{\Gamma\left(\eta+\delta\ell-\nu s+\xi\right)\Gamma\left(\mu+\lambda\ell-\sigma s+\xi\right)\Gamma\left(\mu+\lambda\ell-\sigma s+\xi+\alpha+\alpha'+\beta'-\gamma\right)}{\Gamma\left(\eta+\delta\ell-\nu s\right)\Gamma\left(\mu+\lambda\ell-\sigma s+\xi+\alpha+\alpha'-\gamma\right)\Gamma\left(\mu+\lambda\ell-\sigma s+\xi+\alpha+\beta'-\gamma\right)}$$

$$\times\frac{\Gamma\left(\mu+\lambda\ell-\sigma s+\xi+\alpha-\beta\right)}{\Gamma\left(\mu+\lambda\ell-\sigma s+\xi-\beta\right)}.$$

Finally using $\frac{d^n}{dx^n}x^m=\frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n}$ $(m\geq n)$ and re-interpreting the Mellin-Barnes contour integrals in terms of the ℵ-function of two variables (7), after a little simplification, the $\mathcal{D}_1$ is seen to yield the right-hand side of (30). This completes the proof.

□

In view of the relation (20), we get a (presumably) new formula concerning the left-sided Saigo fractional derivative operator in the following corollary (see [12, 13, 14]).

Corollary 4.2. *Let* α, β, γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\alpha) > 0$. *Also let* λ, $\sigma \in \mathbb{R}^+$. *Then the following formula holds true:*

(33)

$$\Bigg\{D_{0+}^{\alpha,\beta,\gamma}\Bigg(t^{\mu-1}(b-at)^{-\eta}\,S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right] \times \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^{\sigma}(b-at)^{-\upsilon}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},\cdots,[\tau_j(a_j,A_j)]_{n+1,p_k}\\(b_j,B_j)_{1,m},\cdots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right]\Bigg)\Bigg\}(x)$$

$$= b^{-\eta}\,x^{\mu+\beta-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}\,x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right)$$

$$\times\,\aleph_{3,3;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,3;m,n;1,0}\left[\begin{matrix}zb^{-\upsilon}x^{\sigma}\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}V_1';(a_j,A_j)_{1,n},\cdots,[\tau_j(a_j,A_j)]_{n+1,p_k};-\\V_2';(b_j,B_j)_{1,m},\cdots,[\tau_j(b_j,B_j)]_{m+1,q_k};(0,-1)\end{matrix}\right],$$

where V_1' *and* V_2' *are the following horizontal arrays:*

$$V_1' = (1-\eta-\delta\ell;\upsilon,-1),\,(1-\mu-\lambda\ell;\sigma,-1),\,(1-\mu-\lambda\ell-\alpha-\beta-\gamma;\sigma,-1)$$

and

$$V_2' = (1-\eta-\delta\ell;\upsilon,0),\,(1-\mu-\lambda\ell-\beta;\sigma,-1)\;(1-\mu-\lambda\ell-\gamma;\sigma,-1).$$

Also the following conditions are satisfied:

$$\Re(\mu)+\sigma\min_{1\le j\le m}\left(\frac{\Re(b_j)}{B_j}\right)+\max\{0,\Re(\beta),\Re(\alpha+\beta+\gamma)\}>0;$$

$$\Re(\eta)+\upsilon\min_{1\le j\le m}\left(\frac{\Re(b_j)}{B_j}\right)+\max\{0,\Re(\beta),\Re(\alpha+\beta+\gamma)\}>0;$$

and $\left|\frac{a}{b}x\right| < 1$. *Further the other conditions are derivable from those in* (3)-(6).

Further if we set $\beta = -\alpha$ in the result of Corollary 4.2, we obtain a (presumably) new formula concerning the left-sided Riemann-Liouville fractional derivative operator asserted in the following corollary (see [10, 12]).

Corollary 4.3. *Let* α, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\alpha) > 0$. *Also let* λ, $\sigma \in \mathbb{R}^+$. *Then the following formula holds true:*

(34)
$$\begin{aligned}&\left\{D_{0+}^{\alpha}\left(t^{\mu-1}(b-at)^{-\eta}\,S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right]\right.\right.\\
&\quad\left.\left.\times\ \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^{\sigma}(b-at)^{-\upsilon}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k}\\(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right]\right)\right\}(x)\\
&\quad= b^{-\eta}x^{\mu-\alpha-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right)\\
&\times\aleph_{2,2;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,2;m,n;1,0}\left[\begin{matrix}zb^{-\upsilon}x^{\sigma}\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}V_1'';(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k};-\\V_2'';(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k};(0,-1)\end{matrix}\right],\end{aligned}$$

where V_1'' *and* V_2'' *are the following horizontal arrays:*

$$V_1''=(1-\eta-\delta\ell;\upsilon,-1),\,(1-\mu-\lambda\ell;\sigma,-1);$$
$$V_2''=(1-\eta-\delta\ell;\upsilon,0),\,(1-\mu-\lambda\ell+\alpha;\sigma,-1)$$

and $\left|\frac{a}{b}x\right|<1$. *Further the other conditions are derivable from those in* Corollary 4.2.

Theorem 4.4. *Let* α, α', β, β', γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\gamma) > 0$. *Also let* λ, $\sigma \in \mathbb{R}^+$. *Then the following formula holds true:*

(35)
$$\begin{aligned}&\left\{D_{-}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\mu-1}(b-at)^{-\eta}\,S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right]\right.\right.\\
&\quad\left.\left.\times\ \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^{\sigma}(b-at)^{-\upsilon}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k}\\(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k}\end{matrix}\right]\right)\right\}(x)\\
&\quad= b^{-\eta}x^{\mu+\alpha+\alpha'-\gamma-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right)\\
&\times\aleph_{4,4;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,4;m,n;1,0}\left[\begin{matrix}zb^{-\upsilon}x^{\sigma}\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}W_1;(a_j,A_j)_{1,n},\ldots,[\tau_j(a_j,A_j)]_{n+1,p_k};-\\W_2;(b_j,B_j)_{1,m},\ldots,[\tau_j(b_j,B_j)]_{m+1,q_k};(0,-1)\end{matrix}\right],\end{aligned}$$

where W_1 *and* W_2 *are the following horizontal arrays:*

$$\begin{aligned}W_1=&(1-\eta-\delta\ell;\upsilon,-1),\,\left(\mu+\lambda\ell+\alpha+\alpha'-\gamma;-\sigma,1\right),\\&\left(\mu+\lambda\ell+\alpha'+\beta-\gamma;-\sigma,1\right),\,\left(\mu+\lambda\ell-\beta';-\sigma,1\right)\end{aligned}$$

and

$$\begin{aligned}W_2=&(1-\eta-\delta\ell;\upsilon,0),\,(\mu+\lambda\ell;-\sigma,1),\\&\left(\mu+\lambda\ell+\alpha+\alpha'+\beta-\gamma;-\sigma,1\right),\,\left(\mu+\lambda\ell+\alpha'-\beta';-\sigma,1\right).\end{aligned}$$

Also the following conditions are satisfied:

$$\Re(\mu)+\sigma\max_{1\le j\le n}\left(\frac{\Re(a_j)-1}{A_j}\right)<1+\min\left\{\Re(-\beta),\Re\left(\gamma-\alpha-\alpha'-[m/n]\right),\Re\left(\gamma-\alpha'-\beta\right)\right\};$$

$$\Re(\eta)+\upsilon \max_{1\leq j\leq n}\left(\frac{\Re(a_j)-1}{A_j}\right) < 1+\min\left\{\Re\left(-\beta\right), \Re\left(\gamma-\alpha-\alpha'-[m/n]\right), \Re\left(\gamma-\alpha'-\beta\right)\right\};$$

and $\left|\frac{a}{b}x\right| < 1$. *Further the other conditions are derivable from those in* (3)-(6).

Proof. A similar argument as in the proof of Theorem 4.1 will establish the result here. So we choose to skip the detailed account of its proof. □

In view of the relation (21), we get a formula concerning the right-sided Saigo fractional derivative operator given in the following corollary (see [12, 13, 14]).

Corollary 4.5. *Let* α, β, γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ *with* $\Re(\alpha) > 0$. *Also let* λ, $\sigma \in \mathbb{R}^+$. *Then the following formula holds true:*

(36)

$$\Bigg\{D_{-}^{\alpha,\beta,\gamma}\Bigg(t^{\mu-1}\left(b-at\right)^{-\eta} S_n^m\left[t^{\lambda}\left(b-at\right)^{-\delta}\right] \times \aleph_{p_k,q_k,\tau_k;r}^{m,n}\left[z\,t^{\sigma}\left(b-at\right)^{-\upsilon}\,\middle|\, \begin{matrix}(a_j,A_j)_{1,n},\ldots,[\tau_j\left(a_j,A_j\right)]_{n+1,p_k}\\ (b_j,B_j)_{1,m},\ldots,[\tau_j\left(b_j,B_j\right)]_{m+1,q_k}\end{matrix}\right]\Bigg)\Bigg\}(x)$$
$$= b^{-\eta}\,x^{\mu+\beta-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}\,x^{\lambda\ell}\left(\sum_{j=1}^{r}\tau_j'\right)$$
$$\times\,\aleph_{3,3;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,3;m,n;1,0}\left[\begin{matrix}zb^{-\upsilon}x^{\sigma}\\ \left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}W_1';(a_j,A_j)_{1,n},\ldots,[\tau_j\left(a_j,A_j\right)]_{n+1,p_k};-\\ W_2';(b_j,B_j)_{1,m},\ldots,[\tau_j\left(b_j,B_j\right)]_{m+1,q_k};(0,-1)\end{matrix}\right],$$

where W_1' *and* W_2' *are the following horizontal arrays:*

$$W_1' = (1-\eta-\delta\ell;\upsilon,-1),\,(\mu+\lambda\ell+\beta;-\sigma,1),\,(\mu+\lambda\ell-\alpha-\gamma;-\sigma,1)$$

and

$$W_2' = (1-\eta-\delta\ell;\upsilon,0),\,(\mu+\lambda\ell;-\sigma,1)\,(\mu+\lambda\ell+\beta-\gamma;-\sigma,1)\,.$$

Also the following conditions are satisfied:

$$\Re(\mu)+\sigma\max_{1\leq j\leq n}\left(\frac{\Re(a_j)-1}{A_j}\right) < 1+\min\left\{0,\,[\Re\left(\alpha\right)]-\Re\left(\beta\right)-1,\,\Re\left(\alpha+\gamma\right)\right\};$$

$$\Re(\eta)+\upsilon\max_{1\leq j\leq n}\left(\frac{\Re(a_j)-1}{A_j}\right) < 1+\min\,[0,\,\{\Re\left(\alpha\right)]-\Re\left(\beta\right)-1,\,\Re\left(\alpha+\gamma\right)\};$$

and $\left|\frac{a}{b}x\right| < 1$. *Further the other conditions are assumed and derivable from those in* (3)-(6).

Further if we set $\beta = -\alpha$ in (36), we obtain a formula concerning the right-sided Riemann-Liouville fractional derivative operator stated in the following corollary (see [10, 12]):

Corollary 4.6. *Let α, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ with $\Re(\alpha) > 0$. Also let λ, $\sigma \in \mathbb{R}^+$. Then the following formula holds true:*

(37)

$$\Bigg\{ D_-^{\alpha} \Bigg(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \times \aleph_{p_k,q_k,\tau_k;r}^{m,n} \left[z\, t^{\sigma} (b-at)^{-\upsilon} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots, [\tau_j (a_j, A_j)]_{n+1,p_k} \\ (b_j, B_j)_{1,m}, \ldots, [\tau_j (b_j, B_j)]_{m+1,q_k} \end{matrix} \right] \Bigg) \Bigg\} (x)$$

$$= b^{-\eta} x^{\mu-\alpha-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell} \left(\sum_{j=1}^{r} \tau_j' \right)$$

$$\times \aleph_{2,2;p_k,q_k,\tau_k;0,1,\tau_k';r}^{0,2;m,n;1,0} \left[\begin{matrix} z b^{-\upsilon} x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1} \end{matrix} \,\middle|\, \begin{matrix} W_1''; (a_j, A_j)_{1,n}, \ldots, [\tau_j (a_j, A_j)]_{n+1,p_k}; - \\ W_2''; (b_j, B_j)_{1,m}, \ldots, [\tau_j (b_j, B_j)]_{m+1,q_k}; (0,-1) \end{matrix} \right],$$

where W_1'' and W_2'' are the following horizontal arrays:

$$W_1'' = (1-\eta-\delta\ell; \upsilon, -1), (\mu+\lambda\ell-\alpha; -\sigma, 1);$$

$$W_2'' = (1-\eta-\delta\ell; \upsilon, 0), (\mu+\lambda\ell; -\sigma, 1);$$

and $\left|\frac{a}{b}x\right| < 1$. Further the other conditions are assumed and derivable from those of Corollary (4.5).

5. Further Special Cases

The $\aleph$-function reduces to the I-function and the H-function. The H-function reduces to a large number of special functions (see, *e.g.*, [2, 6, 8, 9, 11, 16, 18]). So the main results in Theorems 3.1-4.4 can be further specialized to yield many (presumably) new formulas for fractional calculus involving a number of special functions. Here we specialize only the result in Theorem 3.1 to demonstrate some reduced formulas.

(i) If we put $\tau_j = 1$ $(j \in \overline{1, r})$ in Theorem 3.1, then the Aleph function reduces to the I-function (see [2, 16]; see also Remark 1).

Let α, α', β, β', γ, μ, η, δ, υ, z, a, $b \in \mathbb{C}$ with $\Re(\gamma) > 0$. Also let λ, $\sigma \in \mathbb{R}^+$. Then the following formula holds true:

(38)

$$\Bigg\{ I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma} \Bigg(t^{\mu-1} (b-at)^{-\eta} S_n^m \left[t^{\lambda} (b-at)^{-\delta} \right] \times I_{p_k,q_k;r}^{m,n} \left[z\, t^{\sigma} (b-at)^{-\upsilon} \,\middle|\, \begin{matrix} (a_j, A_j)_{1,n}, \ldots (a_j, A_j)_{n+1,p_k} \\ (b_j, B_j)_{1,m}, \ldots, (b_j, B_j)_{m+1,q_k} \end{matrix} \right] \Bigg) \Bigg\} (x)$$

$$= b^{-\eta} x^{\mu-\alpha-\alpha'+\gamma-1} \sum_{\ell=0}^{[n/m]} \frac{(-n)_{m\ell}}{\ell!} A_{n,\ell}\, b^{-\delta\ell} x^{\lambda\ell}$$

$$\times I_{4,4;p_k,q_k;0,1,;r}^{0,4;m,n;1,0} \left[\begin{matrix} z b^{-\upsilon} x^{\sigma} \\ \left(-\frac{a}{b}x\right)^{-1} \end{matrix} \,\middle|\, \begin{matrix} E_1; (a_j, A_j)_{1,n}, \ldots, (a_j, A_j)_{n+1,p_k}; - \\ E_2; (b_j, B_j)_{1,m}, \ldots, (b_j, B_j)_{m+1,q_k}; (0,-1) \end{matrix} \right],$$

where E_1 and E_2 are the same as given in Theorem 3.1 and the other conditions are assumed and derivable from those in Theorem 3.1.

(ii) If we put $\tau_j = 1$ $(j \in (\overline{1,r})$ in Theorem 3.1 with $r = 1$, then the Aleph function reduces to the H-function (see, *e.g.*, [9]). Then we obtain the following formula:

$$\begin{aligned}&\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\mu-1}(b-at)^{-\eta} S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right]\right.\right.\\&\qquad\left.\left.\times H_{p,q}^{m,n}\left[z\,t^{\sigma}(b-at)^{-\upsilon}\,\middle|\,\begin{matrix}(a_j,A_j)_{1,p}\\(b_j,B_j)_{1,q}\end{matrix}\right]\right)\right\}(x)\\&=b^{-\eta}x^{\mu-\alpha-\alpha'+\gamma-1}\sum_{\ell=0}^{[n/m]}\frac{(-n)_{m\ell}}{\ell!}A_{n,\ell}\,b^{-\delta\ell}x^{\lambda\ell}\\&\quad\times H_{4,4;p,q;0,1}^{0,4;m,n;1,0}\left[\begin{matrix}zb^{-\upsilon}x^{\sigma}\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}E_1;(a_j,A_j)_{1,p};—\\E_2;(b_j,B_j)_{1,q};(0,-1)\end{matrix}\right],\end{aligned}\tag{39}$$

where E_1 and E_2 are the same as given in Theorem 3.1 and the other conditions are assumed and derivable from those in Theorem 3.1.

(iii) If we use a known relation between Mittag-Leffler function $E_{\beta,\gamma}^{\delta}$ and the H-function (see, *e.g.*, [9, p. 25, Eq. (1.137)]):

$$E_{\alpha,\beta}^{\gamma}(z)=\frac{1}{\Gamma(\gamma)}H_{1,2}^{1,1}\left[-z\,\middle|\,\begin{matrix}(1-\gamma,1)\\(0,1),(1-\beta,\alpha)\end{matrix}\right]\quad(\Re(\gamma)>0)\tag{40}$$

in (39), we obtain the following formula:

$$\begin{aligned}&\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\mu-1}(b-at)^{-\eta} S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right]E_{e,f}^{\rho}\left[z\,t^{\sigma}(b-at)^{-\upsilon}\right]\right)\right\}(x)\\&=b^{-\eta}x^{\mu-\alpha-\alpha'+\gamma-1}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,b^{-\delta k}x^{\lambda k}\\&\quad\times\frac{1}{\Gamma(\rho)}H_{4,4;1,2;0,1}^{0,4;1,1;1,0}\left[\begin{matrix}-zb^{-\upsilon}x^{\sigma}\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}E_1;(1-\rho,1);—\\E_2;(0,1),(1-f,e);(0,-1)\end{matrix}\right],\end{aligned}\tag{41}$$

where E_1 and E_2 are the same as given in (22).

(iv) If we use a known relation involving the generalized Wright hypergeometric function ${}_p\Psi_q$ (see, *e.g.*, [9, p. 25, Eq. (1.140)] in (39), we obtain the following formula:

$$\begin{aligned}&\left\{I_{0+}^{\alpha,\alpha',\beta,\beta',\gamma}\left(t^{\mu-1}(b-at)^{-\eta} S_n^m\left[t^{\lambda}(b-at)^{-\delta}\right]\right.\right.\\&\qquad\left.\left.\times {}_p\Psi_q\left[z\,t^{\sigma}(b-at)^{-\upsilon}\,\middle|\,\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right]\right)\right\}(x)\\&=b^{-\eta}x^{\mu-\alpha-\alpha'+\gamma-1}\sum_{k=0}^{[n/m]}\frac{(-n)_{mk}}{k!}A_{n,k}\,b^{-\delta k}x^{\lambda k}\\&\quad\times H_{4,4;p,q+1;0,1}^{0,4;1,p;1,0}\left[\begin{matrix}-zb^{-\upsilon}x^{\sigma}\\\left(-\frac{a}{b}x\right)^{-1}\end{matrix}\,\middle|\,\begin{matrix}E_1;(1-a_p,A_p);—\\E_2;(0,1),(1-b_q,B_q);(0,-1)\end{matrix}\right],\end{aligned}$$

where E_1 and E_2 are the same as given in (22).

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[a]Department of Mathematics & Statistics, Jai Narain Vyas University, Jodhpur 342005, India
E-mail address: dinesh_dino03@yahoo.com

[b]Department of Mathematics & Statistics, Jai Narain Vyas University, Jodhpur 342005, India
E-mail address: drrkbgupta@yahoo.co.in

[c]Department of Mathematics & Statistics, Jai Narain Vyas University, Jodhpur 342005, India
E-mail address: b.s.shaktawatmath@gmail.com

[d]Department of Mathematics, Dongguk University, Gyeongju 38066, Republic of Korea
E-mail address: junesang@mail.dongguk.ac.kr